GCE AS/A Level

0978/01

MATHEMATICS – FP2
Further Pure Mathematics

MONDAY, 26 JUNE 2017 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined on the domain $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ by

$$f(x) = \sec x + x \tan x.$$

Determine whether f is even, odd or neither even nor odd. [3]

2. Evaluate the integral

$$\int_0^2 \frac{2x^2 + 5}{x^2 + 4}\, dx,$$

giving your answer in the form $a + b\pi$, where a, b are constants to be determined. [5]

3. Find the three cube roots of the complex number $-8i$. Give your answers in the form $x + iy$ where x, y are either integers or surds. [8]

4. *(a)* Given that $z = \cos\theta + i\sin\theta$, show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

and find a similar expression for $z^n - \dfrac{1}{z^n}$. [4]

(b) By expanding $\left(z + \dfrac{1}{z}\right)^5$, show that

$$\cos^5\theta = a\cos5\theta + b\cos3\theta + c\cos\theta,$$

where a, b, c are constants whose values should be determined. [5]

(c) Hence evaluate the integral

$$\int_0^{\frac{\pi}{2}} \cos^5\theta\, d\theta \cdot$$ [4]

5. Find the general solution to the equation

$$\cos\theta - \cos5\theta = \sin3\theta.$$ [8]

6. The function f is defined by

$$f(x) = \frac{24x^2 + 31x + 9}{(x+1)(2x+1)(3x+1)}.$$

(a) Express $f(x)$ in partial fractions. [4]

(b) (i) Evaluate the integral

$$\int_0^2 f(x)\,dx,$$

giving your answer as $\ln N$, where N is a positive integer.

(ii) Explain briefly why the integral

$$\int_{-2}^0 f(x)\,dx$$

cannot be evaluated. [5]

7. (a) The point $P(x, y)$ moves in such a way that its distance from the point $(a, 0)$ is equal to its distance from the line $x = -a$. Show that the locus of P is the parabola with equation $y^2 = 4ax$. [3]

(b) Determine the equation of the normal at the point $(at^2, 2at)$ on the parabola. [4]

(c) This normal intersects the parabola again at the point $(as^2, 2as)$. Obtain an expression for s in terms of t. [5]

8. The function f is defined by

$$f(x) = x + 3 + \frac{1}{x+1}.$$

(a) Find the equation of

(i) the vertical asymptote on the graph of f,

(ii) the asymptote that is not parallel to a coordinate axis. [2]

(b) Find the coordinates of the stationary points on the graph of f. [5]

(c) (i) Obtain an expression for $f''(x)$.

(ii) Hence classify each of the stationary points as a maximum or a minimum. [3]

(d) Sketch the graph of f, including the asymptotes. [3]

(e) The set S is given by $S = [4, 5]$. Determine $f^{-1}(S)$. [4]

END OF PAPER